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THE INFLUENCE OF REINFORCING STEEL BARS ON THE APPARENT LONGITUDINAL ULTRASONIC PULSE VELOCITY IN CONCRETE: A SIMPLE MATHEMATICAL MODEL.

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Low ultrasonic frequencies must be used in non-destructive testing of heterogeneous materials, such as plain and reinforced concrete, cast iron or timber. When frequencies are low enough, size and shape of tested bodies may influence the measured longitudinal pulse velocities.
An approach which is named "quasi-fluid" is applied to discuss the longitudinal pulse propagation in reinforcing steel bars embedded in concrete. Wave propagation is described by a spatially-averaged dilatational field.
An approximate formula is obtained which relates group velocity with an effective radius of the embedded bars, with transducer frequency, and with the asymptotic P-wave velocity in steel and concrete, amongst other parameters. The evolution of a typical ultrasonic pulse is described using stationary-phase methods.
Several known experimental facts are thus explained.
Finally, common-use empirical correlations between the measured longitudinal pulse velocity and the diameter of embedded steel bars are discussed using the model developed in this paper.

Key words: Ultrasounds, Wave propagation, Non destructive testing,Concrete.

1.INTRODUCTION

Ultrasonic testing of concrete is currently used for quality assessment in large concrete structures cast on site and in mass-produced prefabricated units.
The intrinsic inhomogeneity of concrete limits the carrier frequencies that can be used to 250 kHz and less (being 54 kHz the most common frequency).
Besides, it usually demands two ultrasonic probes (transducers), one for pulse emission and the other for pulse reception. In practice, probes for ultrasonic testing of concrete are not much larger (50 mm) than the customary ones for metals. The contact face of a common cylindrical probe, whose diameter is of the order of magnitude of the

wavelength, radiates besides longitudinal waves, also shear and surface waves of appreciable intensity.

The absence of any directional effect in the probes, as well as the multiple scattering of the waves inside concrete, makes it possible, (in principle) to couple directly any two points on the surface of the specimen being tested.

The fastest ultrasonic signal received in this way is then always a direct longitudinal wave-pulse. This is followed by shear and surface wave-pulses, and by reflected longitudinal waves. For quality tests on concrete the longitudinal acoustic velocity is determined measuring the distance between probes and the time that the direct longitudinal pulse takes to go from the emitting to the receiving probe. A higher velocity usually means a better concrete quality (strength, durability and dimensional stability).

Reinforcement, if present, should be avoided. Indeed, considerably uncertainty is introduced by the higher velocity of pulses in steel and by possible compaction shortcomings in heavily reinforced regions. If the reinforcing bars run in a direction at right angles to the pulse path and the sum of bar diameters is small in relation to path length, the effect is generally not significant. But if the reinforcing bars lie along or parallel to the path of the pulse, the effect may be very significant. It is not only the measurements taken along the reinforcing bars that are affected, but also those in the neighbourhood of the bars. The first pulse taking and indirect path via the reinforcing bar may reach the receiving probe before that going along the direct path through concrete, resulting in a shorter apparent transit time. Now, a relatively small difference in pulse velocity usually corresponds to a relatively large difference in the quality of concrete. So, if reinforcement cannot be avoided, it may be necessary to make some corrections in order to estimate the true longitudinal pulse velocity in bulk concrete.

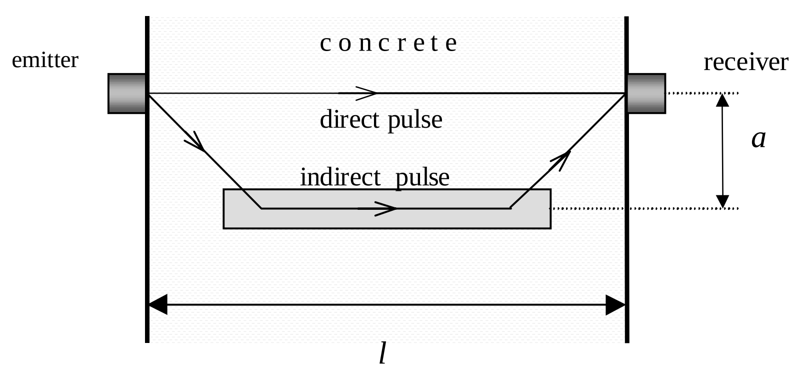

Fig. 1

In BS 4408 pt5, the following correction is recommended for the situation depicted in fig 1.(Bungey,1982).If t is the measured travel time of the pulse, l the apparent path length, V_S is the pulse velocity in steel, and a is the distance shown in the figure, then the true pulse velocity in bulk concrete, V_c, is given by

$$V_c = \frac{2aV_S}{\sqrt{4a^2 + (tV_S - l)^2}} \qquad [1]$$

The apparent pulse velocity in bulk concrete would be

$$V_{c,app} \equiv \frac{l}{t} \qquad [2]$$

The correction may be applied if $V_c \leq V_s$ and if

$$\frac{a}{l} \leq \frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s - V_c}} \qquad [3]$$

The effect of the reinforcement disappears if

$$\frac{a}{l} > \frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s + V_c}} \qquad [4]$$

The aforementioned correction formula is based in the acoustic ray theory of the old refraction method of the seismologists (Pilant,1979).In this case we have an elastic layer of thickness a over an elastic half-infinite bed. P waves are produced by an emitter at the upper face of the layer, and the waves are detected by a receiver, also located at the upper face (Fig.2).

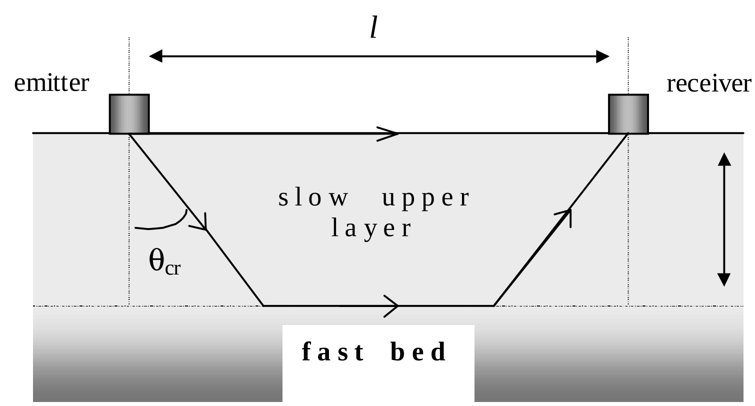

Fig. 2

If the bulk P wave velocity in the bed (V_{L2}) is higher than the bulk P wave velocity in the layer (V_{L1}), the method may be applied to determine a, V_{L1} and V_{L2} varying l and measuring the corresponding travel times t. If $\frac{a}{l}$ is greater than the threshold value

$$\frac{1}{2}\sqrt{\frac{V_{L2} - V_{L1}}{V_{L2} + V_{L1}}} \qquad [5]$$

the direct P wave arrives first. But if $\frac{a}{l}$ is less than this threshold, an indirect P wave arrives first. This wave is totally reflected at the interface. (All the other indirect P waves arrive later). If the layer is a layer of concrete and the bed is a steel bed, V_{P1} would be V_c and V_{P2} would be V_S = 5.90 km/sec. However, Chung (1978) has shown that for pulses travelling in the direction of the axis of the reinforcing bars through a steel-concrete medium, the effective pulse velocity V_e is less than the above mentioned value for P waves in bulk steel and it is influenced by bar diameter according to the approximate empirical formula:

$$V_e = 5.90 - \frac{5.2}{R}\left(5.90 - V_c\right) \qquad [6]$$

Here R is bar´s radius (in mm) and V_c is the true velocity in concrete (in km/sec.) (R must be greater than 5.2 mm. Chung´s formula was obtained for pulses of 54 kHz).
As a consequence, it seems that V_e must be used instead of V_S in formula (1), giving an implicit relation for V_c. However, even now there is controversy about the meaning and the feasibility of this corrections.(The first critical discussion seems to be that of Bungey,1984).
The purpose of the present work is to discuss some aspects of the longitudinal pulse propagation in reinforcing steel bars, with the help of a fairly simple mathematical model.
Even if enough spatial symmetry is assumed, the problem of pulse propagation in steel bars embedded in concrete is fairly difficult to solve using directly an ab-initio numerical analysis of the equations of elastodynamics (Bellomo and Preziosi,1995;Conca and Gatica,1997). So, a simplified analytical approach, as intended in this paper, may be of interest, both for the design of complex digital simulations and for the interpretation of the experimental results obtained with non-destructive testing (ultrasonic) equipments.

2.THE ALMOST-FLUID APPROACH TO LONGITUDINAL PULSE VELOCITY AND THE EQUIVALENT LEAKY WAVEGUIDE.

Let us consider first a steel bar in air. It is a solid wave guide with free (zero-stress) boundary conditions. From the equations of elastodynamics written in cylindrical coordinates (Pochhammer´s equations), and assuming enough symmetry to exclude solutions that correspond to flexural or to torsional vibrations (Kolsky,1964), a numerable infinity of longitudinal propagation modes is obtained (for an infinite waveguide). Each mode is given by a definite dispersion relation between angular frequency ω and wave number k along the waveguide axis, and by a certain modal field of vector displacements in any cross section of the bar.This field is a function of k, and k is an increasing function of ω.
With the exception of the lowest order mode, all the others show a low-frequency cut-off. If ω is below mode´s cut-off frequency, k is imaginary and the mode is non-propagating.When k approaches zero,the phase velocity ω/k for the lowest mode approaches the extensional wave velocity, while the higher modes are non-propagating.When k grows without bound the phase velocity of the lowest mode tends to the velocity of Rayleigh´s surface waves and the corresponding displacement field concentrates in a thin layer adjacent to the boundary of the bar (skin effect).When k grows,the phase velocity of the next higher modes tends to the shear wave velocity.The phase velocity of the highest order modes tends to the dilatational (P) wave velocity.
All this stemmed from numerical analysis of Pochhammer´s analytic solutions to the equations of elastodynamics (Auld,1973).
Now, the conditions at the emitting probe are never such us to initiate precisely the waves indicated by the Pochhammer solutions.As the wave train progresses along the

bar,disturbances along the boundary are produced by reflection and mode conversion so as to reduce there the stress to zero.As a consequence,at the receiving probe a complex time signal is obtained corresponding to a longitudinal wave packet arriving first followed by several trailing pulses.The experimental results suggest that, as a first approximation when kR is a big number, the dilatational component may be considered to be guided along the bar much as in the case of a fluid wave-guide that releases pressure at its boundary (Mc Skimin,1956).But in the solid wave guide case,energy is continually being drained off through mode conversion processes at the boundary.Then,at any cross section of the bar the leading pulse can be build up from the Fourier spectrum of the emitted pulse using only the dilatational part of the Pochhammer solutions.

This is the almost-fluid approximation,which is very useful to consider the effects of the geometric dispersion when the wavelengths are small relative to the smallest dimension of the solid body perpendicular to the direction of pulse propagation (Suárez Antola,1990).The displacement field imposed by the emitting transducer excites mainly the first mode of the almost fluid waveguide,whose group velocity is given by

$$V_g = V_L \sqrt{1 - \frac{\omega_{MS}^{\ 2}}{\omega^2}} \qquad [7]$$

Here V_L is P-wave velocity in steel and

$$\omega_{MS} = \frac{2.40 * V_L}{R} \qquad [8]$$

is a cut-off frequency. We propose to call it "Mc Skimin frequency".(2.40 is the first zero of $J_o(x)$, the Bessel function of the first kind and order zero). This formula can be applied when kR is much greater than one, that is, for high frequencies. But for the low frequencies used in ultrasonic testing of concrete the wavelengths $\lambda = 2\pi/k$ are not small in comparison with bar radius R.(for 54 kHz in steel, $\lambda = 0.11$ m). So, even if the spectra of short ultrasonic pulses of 54 kHz of carrier frequency have significant components at higher frequencies (above 150 kHz), it is not at all clear whether we can study the travel time of the leading dilatational pulse using the almost-fluid approach.

Furthermore, our objetives are in steel bars embedded in concrete. As $V_L´$ (P wave velocity in concrete) and $V_T´$ (S wave velocity in concrete) are less than V_L and V_T in steel, there is no possibility of total reflection when a wave comes to the steel-concrete interface from the steel side. Then a dilatational pulse that travels along an embedded reinforcement bar is continuously loosing energy towards the adjacent concrete. Once the waves go through the steel-concrete interface towards the concrete, they scatter, reflect and convert modes many times at the interfaces between the aggregates and the cement paste. Part of this energy comes back and now part of it can be trapped by total reflection in the concrete layer adjacent to the steel bar. A certain dilatational wave field is thus produced, surrounding the bar and inside the bar, moving along the bar axis and tending fairly quickly to zero as the distance from the bar axis grows. Our suggestion now is to model the propagation of the leading longitudinal pulse (whose travel time is measured in non-destructive testing) using an equivalent leaky waveguide, coaxial with the steel bar, but with an effective radius R_e, greater than bar´s radius R. This equivalent waveguide will be in principle a solid one. However, to simplify our analysis, let us suppose for a moment that it can be treated as almost-fluid. Then, for the first

propagation mode we can apply formula [7] for the group velocity, but with R_e instead of R in the equation [8] for Mc Skimin frequency. The radial dependence of the dilatational field for this mode is given by $J_o(2.40r / R_e)$, where r is the distance from bar´s axis. Now, we can write approximately: $J_o(x) = 1- (x^2/(2.40)^2)$.
Introducing the parameter

$$\theta = J_o\left(2.40\frac{R}{R_e}\right) \qquad [9]$$

we have

$$R_e \approx \frac{R}{\sqrt{1-\theta}} \qquad [10]$$

In general, besides the wave field that propagates through the equivalent leaky waveguide, there is a direct longitudinal pulse that travels with velocity V_L' through bulk concrete. When the radius of the bar is

$$R_o = \frac{2.40*\sqrt{1-\theta}*V_L}{\omega*\sqrt{1-\left(\frac{V_L'}{V_L}\right)^2}} \qquad [11]$$

we have $V_g = V_L'$. If R is greater than R_o , the first pulse that arrives to the receiving transducer would be the leaky waveguide pulse. If R is less than R_o , it would be the direct pulse that travels with velocity V_L'. Then, the effective velocity V_e of longitudinal pulses in reinforced concrete would be given by equation [7] with R_e instead of R, if $R_0 \le R < +\infty$,and by $V_e = V_L'$ if $R \le R_0$ .But the formula for group velocity in the leaky waveguide can be fairly well approximated by

$$V_e \approx V_L - (V_L - V_L')\frac{R_0}{R} \qquad [12]$$

Both give the value V_L' when $R=R_o$, both grow monotonically with R in a hyperbolic fashion and both tend to V_L when R tends to infinity.
Formula [12] jointly with formula [11] constitute a theoretical counterpart of Chung´s empirical formula (equation [6]). Equation [6] was obtained adjusting parameters to the measurements taken from concretes with six different values of the bulk velocity V_L'.
As the empirical value for R_o doesn´t seem to depend of V_L' , from the theoretical formula for R_o (equation [11]) we find that $\theta \approx \left(\frac{V_L'}{V_L}\right)^2$. Then

$$R_0 \approx \frac{2.40*V_L}{\omega} \qquad [13]$$

and

$$R_e = \frac{R}{\sqrt{1-\left(\frac{V_L'}{V_L}\right)^2}} \qquad [14]$$

If the steel bar is in void ($V_L' = 0$), we obtain $R_e=R$ as it should be. If V_L' approaches to V_L, R_e tends to infinity. This means that J_0 (2.40r / R_e) will tend to one for every r and so we will have homogeneous plane waves moving without geometric dispersion, also as it should be.

Now let us put a transducer to measure,at a certain cross-section of the waveguide,a propagating wave packet.The probe produces a scalar (voltage) time signal after making a certain weighted average of the elastodynamic field.So,it will be enough to consider the propagation of a representative scalar field (for example, the average displacement) given by:

$$u(t,z) = \int_{-\infty}^{+\infty} A(\omega) e^{i\phi(\omega)} e^{-\alpha(\omega)z} e^{i(\omega t - k(\omega)z)} d\omega \qquad [15]$$

Here $\alpha(\omega)$ is an attenuation coefficient related with the dilatational energy drainage off the equivalent waveguide, by mode conversion and by leakages towards the bulk of concrete.(We are not considering the energy dissipation due to internal friction or coupled elastic and thermal effects). By analogy with the bar in air case (Suárez Antola,1998), it is possible to guess that

$$\alpha(\omega) \approx \frac{V_L g\left(\frac{R_e \omega}{V_L}\right)}{\omega R_e^{\,2}} \qquad [16]$$

where g(x) is a slowly varying function of its argument. When t is big enough, the method of stationary phase (Segel, 1987; Pilant, 1979) can be applied to the evaluation of the integral [15]. For a given t and z, the main contribution to the integral comes from the frequencies $\omega = \omega_e(t,z)$ that verify

$$\frac{\partial k}{\partial \omega}(\omega_e) = \frac{t}{z} \qquad [17]$$

.

If [17] doesn´t have real roots, u(t,z) is negligible. In our case, from [7] and [17] it follows that

$$\omega_e = \frac{\omega_{MS}}{\sqrt{1-\left(\frac{z}{V_L t}\right)^2}} \qquad [18]$$

so that we have a single real root if z is less than V_Lt. (For the equivalent leaky waveguide, R_e must be put instead of R in the expression [8] for ω_{MS}).

The scalar field [15] can be approximated by (Segel, 1987; Pilant, 1979) :

$$u(t,z) \approx \frac{2e^{-\alpha(\omega_e)z} A(\omega_e)}{\left[\frac{1}{2\pi}\left|\frac{\partial^2 k}{\partial \omega^2}(\omega_e)\right| z\right]^{1/2}} \cos\left(\omega_e t - k(\omega_e)z + \phi(\omega_e) + s\frac{\pi}{4}\right) \qquad [19]$$

where s is the sign of $\frac{\partial^2 k}{\partial \omega^2}(\omega_e) \neq 0$.

Formula [19] is a local approximation by an harmonic wave of frequency ω_e and wave number $k(\omega_e)$.

But when z/t varies, ω_e changes as well (according to formula [18]),so that we obtain a wave modulated in amplitude,frequence and phase.

The amplitude spectrum $A(\omega)$ depends of the emitted pulse. Consider an incoming pulse $u(t,0) = A_o e^{-\mu t}(\mathrm{sen}\,\omega_0 t)H(t)$ (ω_0 is the carrier frequency and H(t) is Heavside´s unit step function).Then we have

$$A(\omega) = \frac{\omega_0}{\sqrt{\left(\mu^2 + {\omega_0}^2 - \omega^2\right)^2 + 4\mu^2\omega^2}} \qquad [20]$$

If the pulse is a short one, as in ultrasonic non-destructive testing, we can put $\mu \approx \omega_0/2\pi$. So, even if we have spectral components of every possible frequency, their amplitudes tend to zero as $1/\omega^2$, when ω grows to infinity. Let $\omega_{e,u}$ be the biggest frequency whose amplitude is over the detection threshold.Then from equation [17] it follows that for a certain position z of the receiving transducer,the first signal detected would be registered in an instant t such that

$$\frac{\left(z/t\right)}{V_L} = \sqrt{1 - \frac{{\omega_{MS}}^2}{{\omega_{e,u}}^2}} \qquad [21]$$

This would give us the apparent dilatational pulse velocity, measured with a propagating wave-packet of a given spectrum of Fourier components.

The effective longitudinal pulse velocity V_e would be given by formula [7], but for $\omega = \omega_{e,u}$ instead of the carrier frequency ω_0 of the emitted pulse. Furthermore,we must use $\omega_{e,u}$ instead of ω_0 in formula [13] for the radius R_o of the bar such that for it $V_e = V_L'$.

3.DISCUSSION AND CONCLUSIONS.

In order to describe the propagation of the leading dilatational pulse in a reinforcement bar embedded in bulk concrete, we introduced an equivalent solid waveguide of radius R_e (given by formula [14]), greater than the geometric radius of the bar. When the number $R_e\omega / V_L$ is big enough (that is,for frequencies high enough) this solid waveguide behaves as almost fluid, in the following sense. Due to interference from waves produced at interfaces,both the axial and the radial displacement fields,as functions of the distance from bar axis,show small oscillations superposed on a fairly smooth trend.If we make an average of these fields over a suitable chosen (and movable) plane región, we obtain a smoothed axial field that seems to obey very approximately a leaky classical wave equation,with velocity V_L and supported by a (fluid) waveguide of radius R_e .If we neglect the influence of the leaks on the wave velocity,we obtain the results stated in section (2).But if $R_e\omega / V_L$ is not big enough, this almost-fluid model cannot be applied and a more realistic model of an equivalent solid waveguide must be constructed. The simplest solid waveguide model can be obtained coupling a purely extensional mode with a suitable shear mode. This, as will be developed elsewhere, gives two emerging modes: a lower extensional-interface mode and an upper shear-dilatational mode. When $R_e\omega / V_L$ increases, this upper mode approaches the first dilatational mode of the almost fluid waveguide.At its turn,the lower mode approaches an interface wave propagation mode,which is very slow and it is less and less excited by the emitting transducer as $R_e\omega / V_L$ increases.

The leading edge of the first dilatational pulse is occupied by the highest frequencies.So we can expect to be able to discuss the measured transit time between emitter and receiver using the almost-fluid approach.

In principle,the spectrum of the incoming pulse has component frequencies as big as we want.As a consequence, a certain fraction of the pulse will always travel with a velocity very near to V_L .But this fraction can be detected,and its corresponding travel time can be measured, only if the amplitude of the cosine in formula [19] is above the detection threshold of the ultrasonic equipment.This is the case when the length l of the trajectory of the pulse is of the same order of magnitude or less than the radius R_e of the equivalent waveguide: here we don´t have geometric dispersion. But when l increases, this high frequency part of the wave-packet will remain under the threshold of detection and the measured transit time will correspond to the travel time of the highest detectable frequency $\omega_{e,u}$. This threshold frequency will depend of l, of the amplitude spectrum of the emitted ultrasonic pulse, of the signal to noise characteristics of the time measuring equipment and of the propagation distortions from the emitter to the receiver described by formula [19]. The fluid waveguide model predicts that $\omega_{e,u}$ should grow if l decreases, if ω_0 (the carrier frequency) increases, and if the total energy of the incoming pulse increases (for the same relative amplitudes).So, if we insist in using the theoretical version of Chung´s formula developed in section (2), from the above remarks and from equation [13] it follows that R_o should decrease if l decreases, or if ω_0 increases,or if the total energy of the pulse increases. From Chung´s experimental value for R_o(0.0052m) and using [13], it results $\omega_{e,u} \approx 14.160/0.0052 = 2.723$ MHz. The carrier frequency is $\omega_0 = 6.28*54$ kHz = 0.3391 MHz so that in this case $\omega_{e,u}$ would be nearly eight times ω_0 .(In Chung´s experiments l was fixed at near 2m).

Using the concept of an equivalent almost-fluid waveguide we have thus explained some known aspects of the geometric dispersion of longitudinal pulses in reinforcement bars. Furthermore, we have seen that several qualitative and quantitative predictions, amenable to experimental testing, stem from the results obtained in section (2).

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